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Keith Daniel Sanborn

MODERN OPTIMAL CONTROL METHODS

APPLIED IN ACTIVE CONTROL OF A

CANTILEVER BEAM IN BENDING VIBRATION.

THESIS

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Captain USAF

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MODERN OPTIMAL CONTROL METHODS
APPLIED IN ACTIVE CONTROL OF A
CANTILEVER BEAM IN BENDING VIBRATION

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

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Keith D. Sanborn

Captain USAF

Graduate Astronautical Engineering

December 1979

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Preface

I would like to express gratitude to my advisor, Dr. R. A. Calico, for his guidance and understanding throughout the preparation of this thesis. I am also indebted to Capt. J. Siverthorn, whose help in understanding certain concepts, most perplexing, was deeply appreciated. Above all, I am indebted to my wife, Rosemary, for her forebearance and encouragement throughout my entire course of study, and to my children, Patrick, Rebecca, and Jessica, for the many times when I was too busy to give them the attention they deserve.

Keith D. Sanborn

Contents

List of Figures

List of Tables

Abstract

The equations of motion for a cantilever beam in bending vibration are developed in state vector form using a normal mode approximation. A linear optimal control system generates a feedback control proportional to the state which is represented by modal amplitudes and velocities determined using position information from sensors. The observer gain matrix and the feedback control gain matrix are both determined from a steady state optimal regulator which minimizes the related quadratic performance index. Control is applied through point force actuators. The effects on beam response of sensor and actuator location, reduced observation and control spillover, restructured control performance index, and a reduced order optimal regulator are examined. Parameter variations of the modal amplitudes were tested to note effects on system stability with both stable and unstable controllers. Singular perturbation theory is used to reconstruct the control performance index, essentially adding a penalty function against any control vector that lies in the suppressed modes subspace. Singular value decomposition is used to construct a transformation matrix through which observation and/or control spillover may be eliminated. A means by which optimal actuator locations may be determined is also presented.

System response is shown to be very sensitive to actuator

location. Singular perturbation provided a method through which control spillover could be minimized for the actuator locations chosen, however, it did not provide the means by which the actuators could be positioned such that the spill-over effect could be eliminated. Optimal actuator locations could be found using singular value decomposition. Control and observation spillover effects could then be completely eliminated only where the number of actuators was equal to or greater than the number of modes to be suppressed. Robustness of system response to modal amplitude errors was very good, and the unstable controllers tested did not appear to seriously affect this robustness.

MODERN OPTIMAL CONTROL METHODS
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Introduction

The control of large space structures has received much attention since the advent of the space shuttle program. Active control of the structural vibrations associated with large structures using modern optimal control methods is one area presently being investigated. The advantages of using these methods lie in the fact that the design process can be fully automated and adapted to a large class of problems rather than a specific model configuration. An optimal number and placement of sensors and actuators could also be determined. Despite these and other advantages, these methods have seen little application in the control of large space structures due to inherent problem areas associated with using them. Two of the main problem areas deal with model reduction and modelling inaccuracies. A very large number of modes may be required to model a flexible structure. Active control of all these modes would be impractical due to computer, sensor, and actuator requirements, therefore a reduced order model of the system must be considered, giving rise to model reduction error. In a complex structure, modelling can be accomplished using finite element approximations, however, this approximation technique produces modal data whose accuracy is uncertain. Modelling inaccuracies cause the modal frequencies and mode shapes to be in error.

Balas (Ref 1) discusses a method by which a class of flexible structures can be controlled through the use of sensors, actuators, a state estimator, and state variable feedback; and he showed that this could lead to closed loop instabilities in the reduced model when modes other than those controlled are considered. These instabilities are caused by the combined effect of what Balas termed observation and control spillover. Sesak (Ref 2) used singular perturbation theory and a highly mathematical approach toward eliminating control spillover. Sesak's technique, however, gave little insight into how or when it might work. Several specific examples were shown, but no general results were obtained. In an attempt to explain the singular perturbation approach, Coradetti presented an analysis of the limiting case of the singular perturbation method and showed that it was equivalent to finding a transformation matrix which reduced control spillover. This transformation matrix can be determined from singular value decomposition of the control matrix associated with those modes in the model that are not actively controlled. Using this technique actuator locations could be predetermined to produce a transformation matrix which would eliminate control spillover. Actuator placement and the number of actuators necessary to achieve this result, however, were not examined in depth; and applying this technique to eliminate observation spillover was not considered.

The purpose of this thesis is to apply Balas' method to a cantilever beam and determine the effect on the beam response of sensor/actuator location, and reduced observation and control spillover using singular perturbation and singular value decomposition techniques. In addition, recent work by Johnson (Ref 5) has shown that standard optimal control design can lead to unstable controllers. The effects of a stable versus an unstable controller on system robustness to modelling inaccuracies, specifically modal error, is also considered.

The approach taken involves forming a discrete system through normal modes approximation. Position sensors are used to determine modal amplitudes, from which a state estimator reconstructs the mode shapes. Control is accomplished using point force actuators through state variable feedback. Both the observer and feedback control use gains produced from steady state optimal regulators. Control performance index restructuring, using singular perturbation, penalizes the control against acting on modes other than those desired. Singular value decomposition of the modal amplitudes at the sensor or actuator location is used to develope a transformation matrix through which observation and/or control spillover may be eliminated.

The main areas examined in this report are: system sensitivity to sensor and actuator placement; reducing observation or control spillover using singular perturbation and singular value decomposition techniques; the number of actuators required and placement necessary to eliminate control spillover; system robustness to modal error for both stable and unstable controllers.

System Model

Equations of Motion

The continuous differential equation of motion for a cantilever beam in bending vibration is defined in Meirovitch (Ref 4:208,209) as

$$\frac{-\partial^2}{\partial x^2} EI(x)\frac{\partial^2 Y(x,t)}{\partial x^2} + f(x,t) = m(x)\frac{\partial^2 Y(x,t)}{\partial t^2} \tag{1}$$

with boundary conditions

$$\begin{aligned} Y(0,t) &= 0 \\ \frac{\partial Y(x,t)}{\partial x}\Big|_{x=0} &= 0 \\ EI(x)\frac{\partial^2 Y(x,t)}{\partial x^2}\Big|_{x=L} &= 0 \\ \frac{\partial}{\partial x} EI(x)\frac{\partial^2 Y(x,t)}{\partial x^2}\Big|_{x=L} &= 0 \end{aligned} \tag{2}$$

where x is the variable along the length of the beam, Y is the vertical deflection, EI(x) is the bending stiffness, f is the applied force, L is the length of the beam, and m is the mass/unit length. The case considered is that of a uniform beam in free vibration so that

$$\begin{aligned} EI(x) &= \text{constant} \\ f(x,t) &= 0 \\ m(x) &= \text{constant} \end{aligned} \tag{3}$$

A discrete form of Eq (1) can be obtained through the substitution

$$Y(x,t) = \sum_{i=1}^{n} \phi_i(x) U_i(t) \tag{4}$$

where ϕ_i are the mode shapes, U_i are the mode amplitudes, and n is the number of modes. An exact solution would require that n equal infinity; however, for practical purposes n is chosen to be much less than infinity. This truncation introduces inaccuracies termed model reduction errors. The above substitution leads to

$$\frac{d^4\phi_i(x)}{dx^4} - \beta_i{}^4\phi_i(x) = 0 \tag{5}$$

where

$$\beta_i^4 = \frac{\omega_i{}^2 m}{EI} \tag{6}$$

with boundary conditions

$$\begin{aligned} &\phi_i(0) = 0 \\ &\left.\frac{d\phi_i(x)}{dx}\right|_{x=0} = 0 \\ &\left.\frac{d^2\phi_i(x)}{dx^2}\right|_{x=L} = 0 \\ &\left.\frac{d^3\phi_i(x)}{dx^3}\right|_{x=L} = 0 \end{aligned} \tag{7}$$

The solution of equation (5) subject to these boundary conditions is

$$\begin{aligned} \phi_i(x) = {} & \frac{(\sin\beta_i L - \sinh\beta_i L)(\sin\beta_i x - \sinh\beta_i x)}{(mL)^{\frac{1}{2}}\sin\beta_i L \sinh\beta_i L} \\ & + \frac{(\cos\beta_i L + \cosh\beta_i L)(\cos\beta_i x - \cosh\beta_i x)}{(mL)^{\frac{1}{2}}\sin\beta_i L \sinh\beta_i L} \end{aligned} \tag{8}$$

where $\beta_i L$ is determined from the characteristic equation

$$\cos\beta_i L \cosh\beta_i L = -1. \tag{9}$$

The mode shapes, ϕ_i, are normalized with respect to the constant mass density, that is

$$\int_0^L m\phi_i(x)\phi_j(x)dx = \delta_{ij} \tag{10}$$

where δ_{ij} is the Kronecker delta. These mode shapes and modal amplitudes are then used to form a linear system model to which modern control theory is applied.

Linear System Model

The number of modes, n, in a structural model may still be very large. For practical purposes the controller should be concerned with as few modes as possible and still maintain a stable system. A determination of which modes need to be controlled will be discussed later, however, for the time being it is accepted that the formulation of a linear model should allow for this. If the model to be developed is considered to consist of three parts, a controlled, a suppressed, and an un-modelled part; Eq (4) can be partioned.

$$Y(x,t) = Y_c(x,t) + Y_s(x,t) + Y_{um}(x,t) \tag{11}$$

Where the suppressed modes are those over which control is unnecessary, however, their behavior must be considered. The un-modelled modes are generally the higher frequency modes where natural damping is condisered adequate to preclude the development of any instabilities.

The controlled part of Eq (11) is

$$Y_c(x,t) = \sum_{i=1}^{C} \phi_i(x)U_i(t) \tag{12}$$

the suppressed part

$$Y_s(x,t) = \sum_{j=c+1}^{c+s} \phi_j(x)U_j(t) \tag{13}$$

and the un-modelled part

$$Y_{um}(x,t) = \sum_{k=c+s+1}^{n} \phi_k(x)U_k(t)$$

where c represents the number of controlled modes, s is the number of suppressed modes, and n is the total number of modes in the model. Further consideration of the un-modelled modes, however, is necessary in that the controller that will be developed in the linear model will have no knowledge of these modes. The purpose of discussing the un-modelled modes is to demonstrate that the model reduction process actually involves two truncations, the first to a finite number of modes to describe a structure, and the second limiting concern only to those considered critical. Both of these truncations lead to inaccuracies.

The amplitudes of the controlled and suppressed modes together with their rates of change form the states

$$\bar{V}_c(t) = \left[U_i^T(t) \ \vdots \ \dot{U}_i^T(t) \right]^T \tag{14}$$

and

$$\bar{V}_s(t) = \left[U_j^T(t) \ \vdots \ U_j^T(t) \right]^T \tag{15}$$

substituting this into the discrete form of the equation of motion, the system can be modelled by

$$\dot{\bar{V}}_c(t) = \underline{A}_c\bar{V}_c(t) + \underline{B}_c\bar{f}(t) \tag{16}$$

$$\dot{\bar{V}}_s(t) = \underline{A}_s\bar{V}_s(t) + \underline{B}_s\bar{f}(t) \tag{17}$$

where a dash over a variable represents a vector, and a dash under a matrix. $\overline{f}(t)$ is the vector of control inputs.

The system parameter matrices are

$$\underline{A}_c = \begin{bmatrix} \underline{0} & \underline{I} \\ -\underline{\Omega}_c & -2\zeta_i\omega_i \end{bmatrix} \tag{18}$$

$$\underline{A}_s = \begin{bmatrix} \underline{0} & \underline{I} \\ -\underline{\Omega}_s & -2\zeta_j\omega_j \end{bmatrix} \tag{19}$$

$$\underline{B}_c = \begin{bmatrix} \underline{0} \\ \underset{\sim}{B}_c \end{bmatrix} \tag{20}$$

$$\underline{B}_s \quad \begin{bmatrix} \underline{0} \\ \underset{\sim}{B}_s \end{bmatrix} \tag{21}$$

where ω_i, ω_j are the natural frequencies of the controlled and suppressed modes respectively, the $\underline{\Omega}$ matrices are diagonal matrices whose elements are the squares of the natural frequencies, ζ is the damping factor, $\underline{0}$ and $\underline{I}$ are the null and identity matrices, $\underset{\sim}{B}_c$ and $\underset{\sim}{B}_s$ are the matrices whose columns are the mode shapes evaluated at the actuator locations, $x_1 \ldots x_a$.

$$\underset{\sim}{B}_c = \begin{bmatrix} \phi_1(x_1)\cdots\phi_1(x_a) \\ \vdots \qquad\qquad \vdots \\ \phi_c(x_1)\cdots\phi\;(x_a) \end{bmatrix} \tag{22}$$

$$\underset{\sim}{B}_s = \begin{bmatrix} \phi_{c+1}(x_1)\cdots\phi_{c+1}(x_a) \\ \vdots \qquad\qquad \vdots \\ \phi_{c+s}(x_1)\cdots\phi_{c+s}(x_a) \end{bmatrix} \tag{23}$$

The sensor output is given by

$$\overline{Y}(t) = \underline{C}_c \overline{V}_c(t) + \underline{C}_s \overline{V}_s(t) \tag{24}$$

$$\underline{C}_c = \left[\underset{\sim}{C}_c \; \vdots \; \underline{0} \right] \tag{25}$$

$$\underline{C}_s = \left[\underset{\sim}{C}_s \; \vdots \; \underline{0} \right] \tag{26}$$

$\underset{\sim}{C}_c$ and $\underset{\sim}{C}_s$ are matrices whose rows are the mode shapes of the controlled and suppressed modes evaluated at sensor locations, $x_1 \ldots x_b$.

$$\underset{\sim}{C}_c = \begin{bmatrix} \phi_1(x_1) \cdots \phi_c(x_1) \\ \vdots \qquad\qquad \vdots \\ \phi_1(x_b) \cdots \phi_c(x_b) \end{bmatrix} \tag{27}$$

$$\underset{\sim}{C}_s \quad \begin{bmatrix} \phi_{c+1}(x_1) \cdots \phi_{c+s}(x_1) \\ \vdots \qquad\qquad \vdots \\ \phi_{c+1}(x_b) \cdots \phi_{c+s}(x_b) \end{bmatrix} \tag{28}$$

The null $\underline{0}$ portion of $\underline{C}_c$ and $\underline{C}_s$ matrices represent the rates of change of the mode shapes at sensor locations and are $\underline{0}$ since displacement only sensors are used. Although this model has been developed for a relatively simple structure using exact mode shapes, the method could easily be used on complex structures using finite element approximations, where no simple continuum description is available.

Since we wish to use state variable feedback, complete knowledge of $\overline{V}_c(t)$ is required, however, the only knowledge available is that contained in the sensor outputs. Therefore, a state estimator is required in order to implement the control system.

Observer Model and

Feedback Control System

The state estimator has the form

$$\dot{\hat{V}}_c = \underline{A}_c \hat{V}_c(t) + \underline{B}_c \overline{f}(t) + \underline{K}\, \overline{Y}(t) - \hat{Y}(t) \qquad (29)$$

where $\hat{V}_c(t)$ is the estimate of $\overline{V}_c(t)$ such that

$$\hat{V}_c(t) = \overline{V}_c(t) + \overline{e}(t) \qquad (30)$$

where $\overline{e}(t)$ is the error, the sensor output is

$$\overline{Y}(t) = \underline{C}_c \overline{V}_c(t) + \underline{C}_s \overline{V}_s(t) \qquad (31)$$

The output corresponding to the estimated state is

$$\hat{Y}(t) = \underline{C}_c \hat{V}_c(t) \qquad (32)$$

Using equations (29), (30), (31), (32) and (16) it can be shown that

$$\dot{\overline{e}}(t) = (\underline{A}_c - \underline{K}\, \underline{C}_c)\overline{e}(t) + \underline{K}\, \underline{C}_s \overline{V}_s(t) \qquad (33)$$

The observer gain matrix, $\underline{K}$, must be developed such that this estimator error decays more rapidly than the system dynamics for proper system response. The eigenvalues of $(\underline{A}_c - \underline{K}\, \underline{C}_c)$ determine the decay rate. Since these eigenvalues are the same as the eigenvalues of $(\underline{A}_c{}^T - \underline{C}_c{}^T \underline{K}^T)$, the equations of motion for the sensing system can be modelled in the form

$$\dot{\overline{w}}(t) = \underline{A}_c{}^T \overline{w}(t) - \underline{C}_c{}^T \overline{g}(t) \qquad (34)$$

where

$$\overline{g}(t) = \underline{K}^T \overline{w}(t) \qquad (35)$$

The observer gain matrix, $\underline{K}$, is then calculated using a steady state optimal regulator, where

$$J = \tfrac{1}{2} \int_0^\infty \overline{w}^T \underline{Q}_s \overline{w} + \overline{g}^T \underline{R}_{ob} \overline{g}\, dt \qquad (36)$$

is minimized. The optimal solution is given by

$$\underline{K}^T = -\underline{R}_{ob}^{-1}\underline{C}_s\underline{P} \tag{37}$$

where $\underline{P}$ is the solution to the algebraic matrix Riccati equation

$$\underline{P}\,\underline{A}_c^T + \underline{A}_c\underline{P} - \underline{P}\,\underline{C}^T\underline{R}_{ob}^{-1}\underline{C}\,\underline{P} + \underline{Q}_s = \underline{0} \tag{38}$$

where $\underline{R}_{ob}$ and $\underline{Q}_s$ are weighting matrices.

Similarly, the control feedback gain matrix, G, is calculated using the steady state optimal regulator

$$J = \tfrac{1}{2}\int_0^\infty (\overline{V}_c^T\underline{F}\overline{V}_c + \overline{f}^T\underline{R}\overline{f})\,dt \tag{39}$$

where $\underline{F}$ and $\underline{R}$ are weighting matrices, and

$$\overline{f}(t) = \underline{G}\hat{V}_c(t)$$

The optimal solution is given by

$$\underline{G} = -\underline{R}^{-1}\underline{B}_c^T\underline{S} \tag{40}$$

where $\underline{S}$ solves the matrix Riccati equation

$$\underline{S}\,\underline{A}_c + \underline{A}_c^T\underline{S} - \underline{S}\,\underline{B}_c\underline{R}^{-1}\underline{B}_c^T\underline{S} + \underline{F} = \underline{0} \tag{41}$$

using these results the equations of motion for the system are now

$$\dot{\overline{V}}_c(t) = (\underline{A}_c + \underline{B}_c\underline{G})\overline{V}_c(t) + \underline{B}_c\underline{G}\,\overline{e}(t) \tag{42}$$

and

$$\dot{\overline{V}}_s(t) = \underline{A}_s\overline{V}_s(t) + \underline{B}_s\underline{G}\,\overline{V}_c(t) + \underline{B}_s\underline{G}\,\overline{e}(t) \tag{43}$$

By defining a composite system state vector

$$\overline{Z}(t) = \left[\overline{V}_c^T(t)\ \vdots\ \overline{e}^T(t)\ \vdots\ \overline{V}_s^T(t)\right]^T \tag{44}$$

a closed loop system model with state variable feedback containing the effects contributed by suppressed modes can be written as

$$\dot{\bar{Z}}(t) = \begin{bmatrix} \underline{A}_c + \underline{B}_c\underline{G} & \underline{B}_c\underline{G} & \underline{0} \\ \underline{0} & \underline{A}_c - \underline{KC}_c & \underline{KC}_s \\ \underline{B}_s\underline{G} & \underline{B}_s\underline{G} & \underline{A}_s \end{bmatrix} \bar{Z}(t) \qquad (45)$$

The terms $\underline{K}\ \underline{C}_s$ and $\underline{B}_s\underline{G}$ are the observation and control spillover terms, respectively. It is evident from this equation that the conbination of the two could produce instabilities in the system.

Linear Control System Represented In Block Diagram Form

The equations now governing the system are

$\dot{\bar{V}} = \underline{A}\ \bar{V} + \underline{B}\bar{f}$	state equation	(46)
$\bar{Y} = C\ \bar{V}$	displacement equation	(47)
$\bar{f} = \underline{G}\ \hat{V}_c$	control equation	(48)
$\dot{\hat{V}}_c = \underline{A}_c\hat{V}_c + \underline{B}_c\bar{f} + \underline{K}(\bar{Y}-\hat{Y})$	estimator equation	(49)

Putting the system into block diagram form and separating the plant from the controller yields Figure 1. Johnson, (Ref 7) showed that through manipulation, this diagram can take the form of Figure 2, from which it is easily seen that the transfer function for the controller is

$$\frac{\bar{f}}{Y}(s) = K(SI - A_c - B_cG + KC_c)^{-1}G \qquad (50)$$

If any of the eigenvalues of $(A_c + B_cG - KC_c)$ are positive, the controller would be unstable. It is interesting to note that in a standard optimal design process an unstable controller could unknowingly be developed, which when connected to the plant would still produce a stable system. In addition to the devastating effects this unstable controller could have on a system if it were ever disconnected from the plant, possibly through an electrical interuption, a sensitivity study of system response to parameter variation was conducted for both stable and unstable controllers.

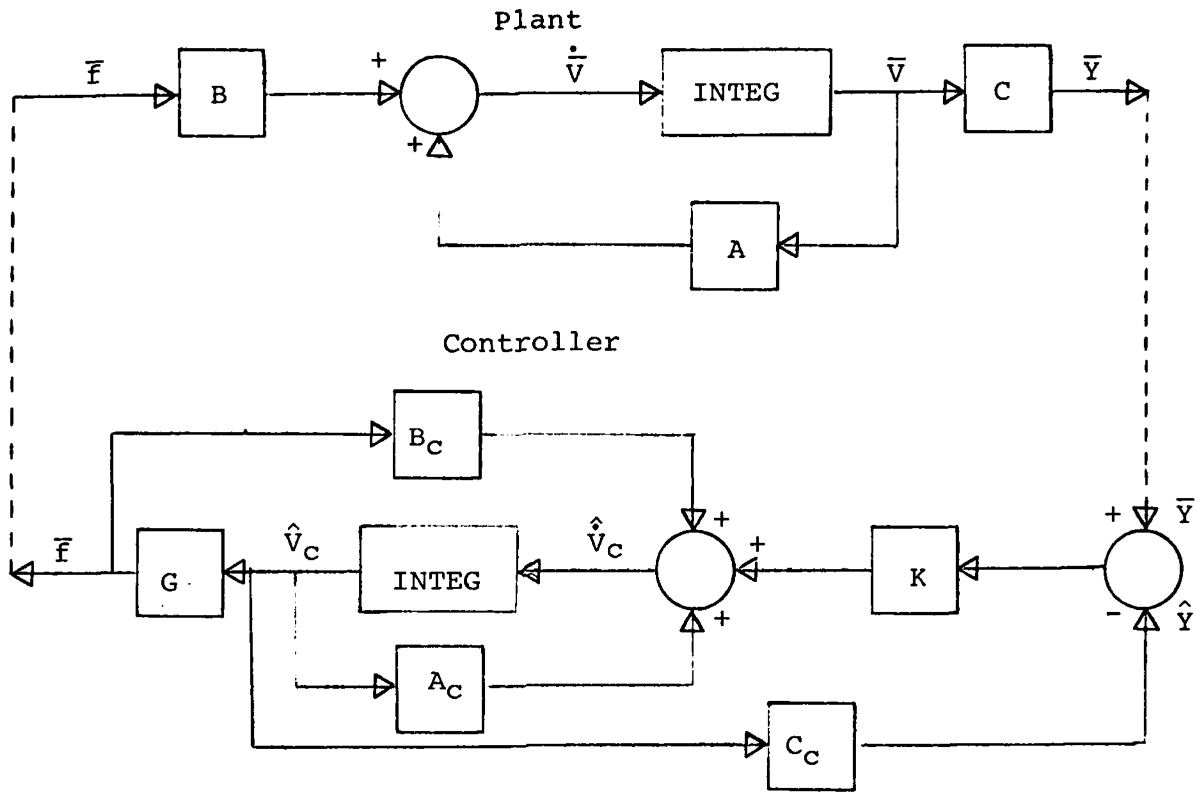

Fig 1. System Represented in Block Diagram Form

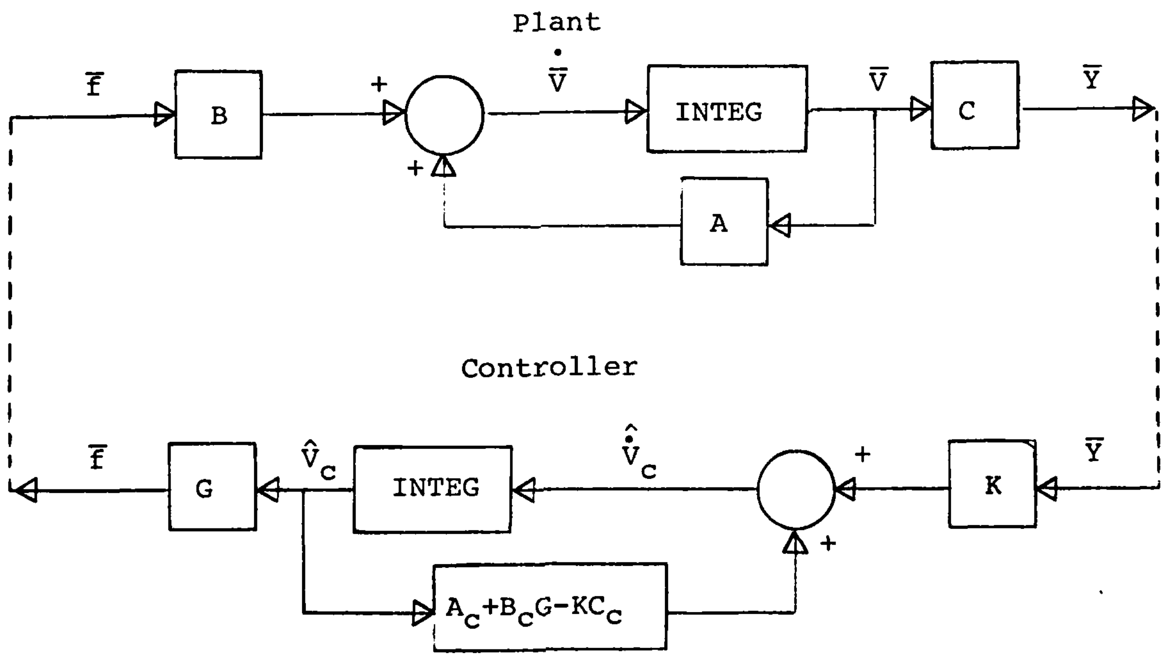

Fig 2. System Represented in Modified Block Diagram Form

Singular Perturbation Optimal Control

This section outlines the procedure developed, by Sesak, of applying singular perturbation theory to the control of a flexible structure. The method is designed to decouple the suppressed and the controlled systems by reducing the signal level in the suppressed portion of the plant. Under these restrictions the controller does not excite the suppressed modes, therefore, the suppressed forced response (control spillover) goes to zero. It can be seen in Eq (45) that if the control spillover is zero, the eigenvalues of the system would be the eigenvalues of the diagonal elements. Since these are designed to be negative the system would then be stable.

Singular perturbation is a variation that changes the order of the differential equation characterizing a particular dynamic system. Consider the performance index of the system under consideration given as

$$J = \tfrac{1}{2} \int_0^\infty (V_c^T F V_c + V_s^T Q_s V_s + f^T R_o f)\, dT \qquad (51)$$

subject to

$$\begin{bmatrix} \dot{V}_c \\ \dot{V}_s \end{bmatrix} = \begin{bmatrix} A_c & 0 \\ 0 & A_s \end{bmatrix} \begin{bmatrix} V_c \\ V_s \end{bmatrix} + \begin{bmatrix} B_c \\ B_s \end{bmatrix} f \qquad (52)$$

It is desired that the dynamics associated with the suppressed modes be stable, therefore, the singular perturbation constraint

$\dot{V}_s \equiv 0$ is used. This yields

$$0 = A_s V_s + B_s f \tag{53}$$

or

$$V_s = -A_a^{-1} B_s f \tag{54}$$

substituting this into Eq (51) yields

$$V_s^T Q_s V_s = f^T B_s^T (A_s^{-1})^T Q_s (A_s^{-1}) B_s f \tag{55}$$

where Q_s is a positive semi-definite weighting matrix. The new performance index is

$$J = \tfrac{1}{2} \int_0^\infty V_c^T F V_c + f^T (R_O \div B_s^T (A_s^{-1})^T Q_s A_s^{-1} B_s) f dt \tag{56}$$

subject to

$$\dot{V}_c = A_c V_c + B_c f \tag{57}$$

The weighting matrix, Q_s, is designed to change the weighting of the penalty on the suppressed modes until control spillover is eliminated. Whether or not this is always possible, or under what conditions it is, was not discussed. Several specific examples were shown, but no general results were obtained. It was found that the method would reduce control spillover, using very large gains, however, the amount of reduction was very dependent on actuator locations.

Transformation Method

A method, presented by Coradetti, for eliminating control spillover, using a transformation matrix, is developed in this section. The transformation matrix, which is determined from singular value decomposition of the suppressed modal amplitudes, is used to constrain $\underline{K}_c$ such that

$$\underline{B}_s \underline{K}_c = \underline{0} \qquad (58)$$

and

$$\underline{B}_c \underline{K}_c \neq \underline{0} \qquad (59)$$

thereby eliminating control spillover, and allowing complete control over the modes chosen. The matrix $\underline{B}_s$ may be expressed through singular value decomposition in the form

$$\underline{B}_s = \underline{U}\ \underline{W}\ \underline{X}^T \qquad (60)$$

where

B_s (m × n) is a rectangular matrix of modal amplitudes evaluated at the actuator locations

U (m × m) is the orthogonal matrix of left singular values

X (n × n) is the orthogonal matrix of right singular values

W (m × n) = $\begin{bmatrix} \underline{\sigma} & \underline{0} \\ \underline{0} & \underline{0} \end{bmatrix}$, where all elements are zero except σ_{ii}, i = 1,...q. These are the non-zero singular values of $\underline{B}_s$. The number q is equal to the rank of $\underline{B}_s$.

Partitioning $\underline{U}$ and $\underline{X}$

$$\underline{U} = \left[U_q \;\vdots\; U_r\right] \tag{61}$$

where $U_q(m \times q)$, $U_r(m \times r)$, $q + r = m$

$$\underline{X} = \left[X_q \;\vdots\; X_p\right] \tag{62}$$

where $X_q(n \times q)$, $X_p(n \times p)$, $q + p = n$

then

$$\underline{B}_s = U_q \sigma X_q^T \tag{63}$$

Defining

$$\underline{L} = X_q \text{ and } \underline{T} = \underline{X}_p \tag{64}$$

and since X is an orthogonal matrix

$$\underline{B}_s\underline{L} = \underline{U}_q\underline{\sigma}\underline{X}_q{}^T\underline{X}_q = \underline{U}_q\underline{\sigma}\ \underline{I} = \underline{U}_q\underline{\sigma} \tag{65}$$

$$\underline{B}_s\underline{T} = \underline{U}_q\underline{\sigma}\underline{X}_q{}^T\underline{X}_p = \underline{U}_q\underline{\sigma}\ \underline{0} = \underline{0} \tag{66}$$

In order to acheive Eq (66) certain conditions have to be met. If the rank of $\underline{B}_s(m \times n)$ is r, then the nullspace formed of solutions to $\underline{B}_s\underline{T} = 0$, has n - r free variables as independent parameters. If r = n, there are no free variables and the nullspace only contains $\underline{T} = \underline{0}$. In the case of r < m there are n - r constraints on $\underline{T}$ in order for $\underline{B}_s\underline{T} = 0$ to be solvable. If one solution exists, then every other solution differs from it by a vector in the nullspace of $\underline{B}_s$. Therefore, in the case of r = n < m the only $\underline{T}$ matrix that is possible for $\underline{B}_s\underline{T} = 0$ is $\underline{T} = \underline{0}$. However, when r < m, n, then there will always be a solution to $\underline{B}_s\underline{T} = \underline{0}$, for which $\underline{T} \neq \underline{0}$. If $\underline{B}_s$ is of full rank, r = min (m,n), and must therefore be reduced to realize Eq (66). Since m is determined by the number of modes suppressed and n by the number of actuators in the system, a $\underline{T}$ matrix can be found which satisfies Eq (66) only when the number of actuators are equal to or greater than the number of modes to be suppressed. The minimum number of actuators, that can be used successfully with this method, is two; since a matrix of rank 1 cannot be reduced.

The reduced order optimal regulator problem is defined by

$$J = \tfrac{1}{2} \int_0^\infty \bar{V}_C{}^T \underline{F} \bar{V}_C + \bar{f}^T \underline{R} \bar{f} \; dT \tag{67}$$

subject to

$$\dot{\bar{V}}_C = \underline{A}_C \bar{V}_C + \underline{B}_C \bar{f} \tag{68}$$

with the additional constraint

$$\underline{B}_S \bar{f} = 0 \tag{69}$$

Defining a new control, $\bar{U}$, where

$$\bar{f} = \underline{T} \bar{U} \tag{70}$$

The new optimal regulator problem is

$$J = \tfrac{1}{2} \int_0^\infty \bar{V}_C{}^T \underline{F} \bar{V}_C + \bar{U}^T \underline{R}_T \bar{U} \; dT \tag{71}$$

subject to

$$\dot{\bar{V}}_C = \underline{A}_C \bar{V}_C + \underline{B}_T \bar{U} \tag{72}$$

where

$$\underline{B}_T = \underline{B}_C \underline{T} \tag{73}$$

and

$$\underline{R}_T = \underline{T}^T \underline{R} \underline{T} \tag{74}$$

The solution is given by

$$\bar{U} = -\underline{K}_T \bar{V}_C \tag{75}$$

$$\underline{K}_T = \underline{R}_T{}^{-1} \underline{B}_T{}^T \underline{S} \tag{76}$$

where $\underline{S}$ is solved from the matrix Ricatti equation

$$\underline{0} = \underline{F} + \underline{S}\,\underline{A}_C + \underline{A}_C{}^T \underline{S} + \underline{S}\,\underline{B}_T \underline{R}_T{}^{-1} \underline{B}_T{}^T \underline{S} \tag{77}$$

The gain matrix is then

$$\underline{K}_c = \underline{TK}_T \tag{78}$$

Actuator locations were determined using the singularity computer program (Appendix B). The program was used to vary actuator locations and determine the nonzero singularities of $\underline{B}_S$. Actuator locations were chosen such that one of these singularities approached zero, indicating a reduction of the rank in $\underline{B}_S$. These actuator locations were then used in the main program and the $\underline{T}$ matrix constructed through singular value decomposition of $\underline{B}_S$ at these locations.

Limit Method

This section contains an extension of Coradetti's Transformation Method which further explains Singular Perturbation Optimal Control. This information was not available durring the investigation conducted on Singular Perturbation Optimal Control and it serves to substantiate the results obtained.

Considering Eqs (75) and (76), the matrix Ricatti Eq (77) can be rewritten as

$$\underline{0} = \underline{F} + \underline{S}\,\underline{A}_c + \underline{A}_c^T\underline{S} + \underline{S}\,\underline{B}_c\underline{T}(\underline{T}^T\underline{R}\,\underline{T})^{-1}\underline{T}^T\underline{B}_c^T\underline{S} \qquad (79)$$

Defining a new matrix, $\underline{H}$, where

$$\underline{H} = \underline{T}(\underline{T}^T\underline{R}\,\underline{T})^{-1}\underline{T}^T \qquad (80)$$

Eq (79) becomes

$$\underline{0} = \underline{F} + \underline{S}\,\underline{A}_c + \underline{A}_c^T\underline{S} + \underline{S}\,\underline{B}_c\underline{H}\,\underline{B}_c^T\underline{S} \qquad (81)$$

The associated performance index is

$$J = \tfrac{1}{2}\int_0^\infty \overline{V}_c^T\underline{F}\overline{V}_c + \overline{f}^T H^{-1}\overline{f}\,dT \qquad (82)$$

subject to

$$\dot{\overline{V}}_c = \underline{A}_c\overline{V}_c + \underline{B}_c\overline{f} \qquad (83)$$

Comparing Eqs (81) and (82) to Eqs (41) and (67), it can be seen that $\underline{H}^{-1} = \underline{R}$ and $\underline{R}^{-1} = \underline{H}$; however, by design, $\underline{T}(\underline{T}^T\underline{R}\,\underline{T})\underline{T}^T$ is not full rank. Therefore, $\underline{H}^{-1}$ does not exist; consequently, there is no finite matrix $\underline{R} = \underline{H}^{-1}$ such that $\underline{R}^{-1} = \underline{H}$, except possibly in a limiting case. Defining a new control weighting matrix, $\underline{R}_s$

$$\underline{R}_s = \underline{R} + a\underline{B}_s^T\underline{Q}\,\underline{B}_s = \underline{H}^{-1} \qquad (84)$$

where $\underline{Q}$ is any positive definite matrix and a is any positive real scalar. Coradetti shows, in an involved mathematical proof, that, in the limiting case of $a \to \infty$, $\underline{R}_s^{-1} = H$. Adding this to the performance index yields

$$J = \tfrac{1}{2} \int_0^\infty \overline{V}_c^T \underline{F} \overline{V}_c + \overline{f}(\underline{R} + a\underline{B}_s^T \underline{Q}\, \underline{B}_s)\overline{f}\, dT \tag{85}$$

It is now quite clear that it is penalizing the performance index against any control vector that lies in the subspace occupied by $\underline{B}_s$. Multiplying $\underline{B}_s^T \underline{Q}\, \underline{B}_s$ by a large enough scalar would effectively eliminate any control acting on the suppressed modes, however, in theory, it would require an infinite value to accomplish this. Coradetti found that this method will decrease control spillover until the numbers involved become so large that computer roundoff errors cause problems in matrix inversion, necessary in solving the matrix Ricatti equation. Singular Perturbation Optimal Control essentially applies the Limit Method with the substitution

$$a\underline{Q} = (\underline{A}_s^{-1})^T \underline{Q}_s A_s^{-1} \tag{86}$$

made in Eq (84). The Limit Method, as presented, shows that while Singular Perturbation Optimal control can reduce control spillover, it can never actually eliminate it.

Computer Model

The function of the main computer program was to assemble the system matrix and determine the associated eigenvalues. The program was modified as needed to incorporate singular perturbation optimal control, the transformation method, and parameter variations. A listing of the program using the transformation Method, and permitting parameter variation is listed in Appendix A.

The first function of the program was to assemble the required matrices. Input data necessary consisted of: beam length, mass, and bending stiffness; sensor and actuator locations; and the roots to the characteristic equation. The root values input were: mode 1 - 1.875, mode 2 - 4.694, mode 3 - 7.855, mode 4 - 10.996, mode 5 - 14.137, mode 6 - 17.278. The beam length, mass, and bending stiffness were 1, and $\underline{F}$, $\underline{F}_{ob}$, $\underline{R}$ and $\underline{R}_{ob}$ weighting matrices were each the identity matrix. The natural frequencies, determined by squaring the roots to the characteristic equation, are then used to construct the $\underline{A}_c$ and $\underline{A}_s$ matrices. Equation (8), for the modal amplitudes, is solved using subroutine Modes (Appendix A), and used to fill matrices $\underline{B}_c$, $\underline{B}_s$, $\underline{C}_c$ and $\underline{C}_s$. Using $\underline{F}$, $\underline{B}_c$, $\underline{A}_c$ and $\underline{R}$, the steady state algebraic Ricatti equation is solved using subroutine MRIC from the Aerospace Medical Research Laboratory Library. The optimal control gain matrix,

$\underline{G}$, is then solved for using Eq (40). The optimal observer gain matrix, K, is solved for in an identical manner. The system matrix, Eq (45), was then constructed, and subroutine EIGRF from the International Mathematical and Statistical Library (IMSL) produced the complex eigenvalues associated with the system. The weighting Matrix, $\underline{Q}_S$, was varied from 1 to 1×10^{14} in examination of the singular perturbation optimal control method. Subroutine LSVDF, from the IMSL library uses singular value decomposition to construct the transformation matrix, $\underline{T}$, from which a reduced order gain was developed.

A second computer program, listed in Appendix B, was used to generate data for determining sensor and actuator placements. In this program sensor or actuator location was varied and the non-zero singularities associated with $\underline{C}_S$ or $\underline{B}_S$ were determined through singular value decomposition. The desired sensor or actuator locations were those which caused one of the non-zero singularities to most closely approximate zero.

The sensitivity study was made using up to ±20% variation of the elements of the $\underline{B}_C$, $\underline{B}_S$, $\underline{C}_C$, and $\underline{C}_S$ matrices. This was accomplished by multiplying the elements of these matrices by one factor, if the sum of its subscripts were even, and by its negative counterpart if the sum was odd. This effectively alters the mode shapes, simulating modal error. Although this method does not account for any frequency errors incurred, the results should essentially reflect the effects produced by modal amplitude error.

Outline of Investigation

An investigation was conducted examining Singular Perturbation Optimal Control and the Transformation Method applied to the model developed of a cantilever beam. In addition, an investigation to determine system robustness to parameter variations was made.

The first area investigated was the use of Singular Perturbation Optimal Control to eliminate control spillover. A single sensor and actuator were initially used to control three modes and suppress one. When it was found, that in the process of reducing the control spillover, control over the first three modes was gradually being lost, another sensor and actuator were added to the system. It was then possible to decrease control spillover and still maintain control. The conditions which determine the reduction realized using this method are discussed.

The second area investigated was the Transformation Method for eliminating control spillover. Two sensors and two actuators were used to control two modes and suppress two. Demand on the system was then increased to control three modes and suppress three. The effectiveness of the method, in eliminating spillover effect and uncoupling system eigenvalues, under these conditions, was examined and some general conclusions were drawn.

The elimination of observation spillover using the Transformation Method was then investigated.

Finally, an investigation of system robustness to parameter variations which simulated modal error, was made. Systems with both stable and unstable controllers were considered.

Investigation

This chapter consists of discussions of the results obtained in investigating the areas presented in the outline.

Singular Perturbation Optimal Control

This section of the investigation applies singular perturbation techniques to reduce control spillover. Initially, a single co-located sensor/actuator pair were used to control the first three modes and suppress the fourth. The weighting matrix, $\underline{Q}_s$, was increased from 10^0 to 10^{14}, and system response was examined. As $\underline{Q}_s$ approached 10^{14}, the real part of the eigenvalue associated with the suppressed mode, which was initally positive, approached zero, indicating the method was reducing control spillover. However, the real part of the eigenvalues associated with the three controlled modes, which were all initially negative, also approached zero, indicating a general loss of control in the system. Since the method did not appear suited to single sensor/actuator operation, two sensors and actuators were then used. In this case control could be maintained, while control spillover was reduced, however, as can be seen in Figure 3, little suppression of the fourth mode is realized until $\underline{Q}_s$ gets very large ($\approx 10^{11}$) as compared to $\underline{R}$ and $\underline{F}$ (≈ 1). After this point, the curve flattens,

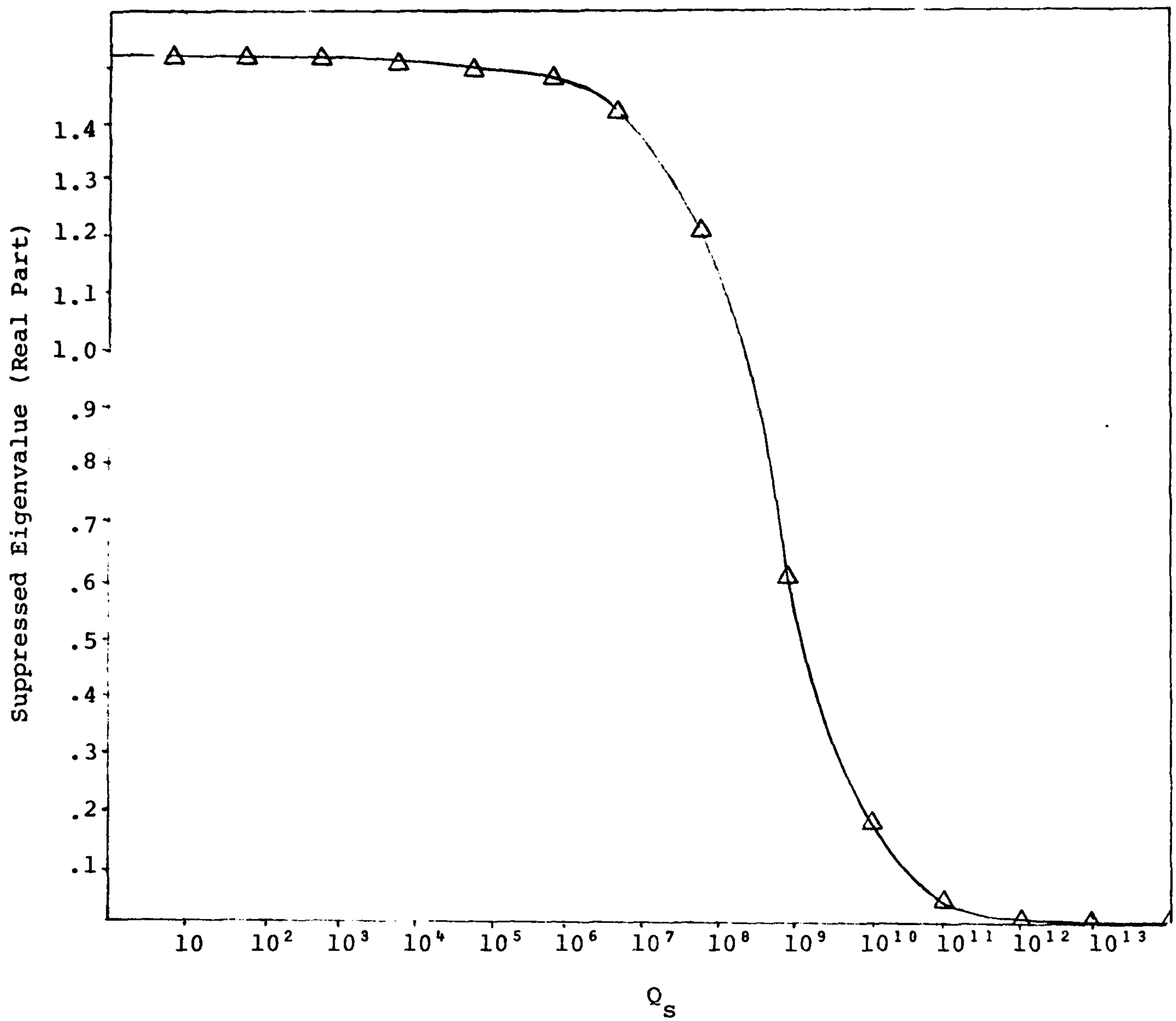

Fig 3. Mode Suppression, Singular Perturbation Optimal Control

and an infinite weighting would theoretically be required to achieve complete suppression. The suppressed mode, in this case, had an eigenvalue of 1.501, $\pm j120.37$ using a $\underline{Q}_s$ of 1, and an eigenvalue of 0.0021, $\pm j120.91$ using a $\underline{Q}_s$ of 10^{14}, which demonstrates the reduction in control spillover achieved using this method. It can be concluded, therefore, that while this method is effective in reducing control spillover, it will never actually eliminate it. The control spillover will be reduced, as $\underline{Q}_s$ increases, only to the point where the numbers involved become so large that computer roundoff error becomes significant. The method also appears to require a reduction in control to reduce control spillover, explaining why control was lost in single actuator operation. These results can be substantiated in light of Coradetti's Limit Method, where computer roundoff error for very large $\underline{Q}_s$ eventually caused problems in matrix inversion, necessary in solving the matrix Ricatti equation.

The Transformation Method

In this section, a transformation matrix is generated to produce a gain matrix for which control spillover is eliminated. Two co-located sensor/actuator pairs were used to control the first two modes and suppress modes three and four. In applying the Transformation Method, $\underline{B}_s$ should be made not full rank. In the case above, the $\underline{B}_s$ matrix has two non-zero singular values usually associated with it, determined from singular value decomposition. Since the number of non-zero singular values of a matrix equals its rank, if one of these can be made approximately zero, through appropriate actuator placement, the rank of $\underline{B}_s$ would decrease from two to one. The Singularity Computer Program (Appendix B) was used to produce Figure 4, from which actuator locations could be determined to decrease the rank of $\underline{B}_s$. In this figure, actuator 1 position is held constant and the minimum singular value of $\underline{B}_s$ is plotted as actuator 2 position is changed. Actuator combinations are chosen such that the minimum singular value approaches zero, excluding co-located actuators, which produce trivial solutions. Two cases are examined in this figure yielding useful actuator combinations of : 0.6, 0.285; 0.6, 0.89; 0.2, 0.585; 0.2, 0.855. The method appeared to work extremely well using these actuator combinations, essentially yielding zero eigenvalues for the two suppressed modes and uncoupling system eigenvalues to six decimal places. Table I demonstrates both of these results. The largest control spillover terms were on the order of 10^{-4}. These terms would decrease as actuator placement approached more precisely the point producing a zero singular value. A comparison between a system utilizing the Transformation Method, and the same system not using it, is presented in Table II, demonstrating the Transformation Method's effectiveness in suppression and uncoupling of system eigenvalues.

Demand on the system was then increased to control the first three modes and suppress modes four, five, and six. As pointed out in the presentation of the Transformation Method, actuator placement, when there are more modes to be suppressed

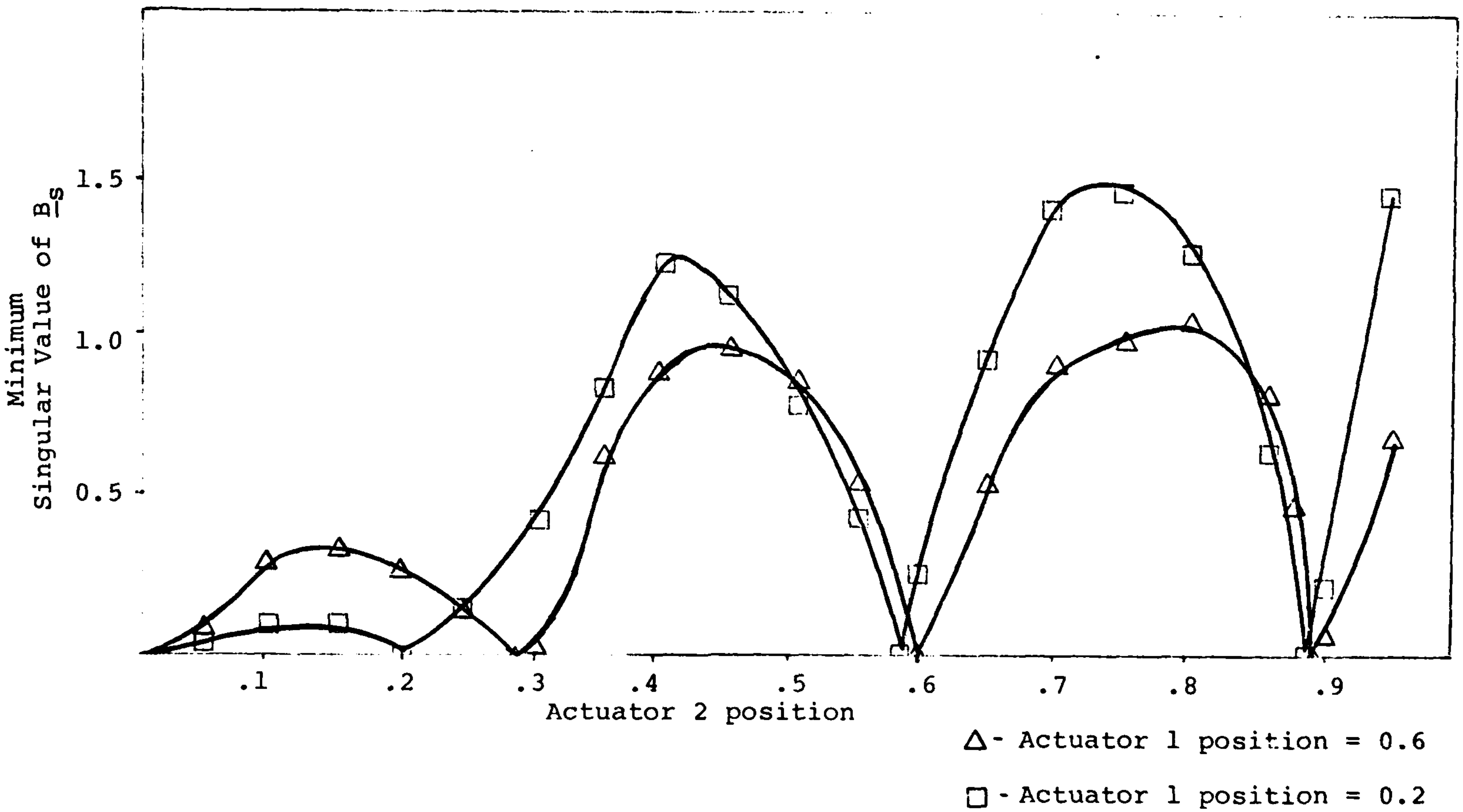

Fig 4. Singular Value of $\underline{B}_s$, 2 Actuators, 2 Suppressed Modes

Table I

Modal Suppression and Eigenvalue Uncoupling

Using the Transformation Method

Eigenvalues of $\underline{A}_c + \underline{B}_c\underline{G}$	-1.341328	0.	(C-1)
	-15.953688	0.	(C-1)
	-5.979912	$\pm j20.493$	(C-2)
Eigenvalues of $\underline{A}_c - \underline{K}\ \underline{C}_c$	-0.923323	$\pm j3.591$	(O-1)
	-13.493944	$\pm j15.910$	(O-2)
Eigenvalues of the System	-2.883098×10^{-7}	$\pm j120.912$	(S-4)
	$+3.058266\times10^{-8}$	$\pm j61.701$	(S-3)
	-1.341328	0.	(C-1)
	-15.953691	0.	(C-1)
	-5.979912	$\pm j20.493$	(C-2)
	-0.923323	$\pm j3.591$	(O-1)
	-13.493942	$\pm j15.910$	(O-2)

Two Controlled, Two Suppressed Modes

Sensor Locations	0.4	0.2
Actuator Locations	0.6	0.285
Singular Values of $\underline{B}_s$	2.12	0.000011

(S-x) - (Suppressed-Mode) Eigenvalues
(C-x) - (Controlled-Mode) Eigenvalues
(O-x) - (Observer-Mode) Eigenvalues

Uncontrolled Structural Damping Ratio = 0.0

Table II

Effectiveness of the Transformation Method

	System Not Using Transformation Method		System Using Transformation Method		
Eigenvalues of $\underline{A}_c + \underline{B}_c\underline{G}$	-1.076042	0.	-1.079741	0.	(C-1)
	-7.720582	±j20.603	-1.379978	±j21.925	(C-2)
	-33.604286	0.	-33.014658	0.	(C-1)
Eigenvalues of $\underline{A}_c - \underline{K}\ \underline{C}_c$	-0.923323	±j3.591	-0.923314	±j3.591	(O-1)
	-13.493940	±j15.910	-13.493837	±j15.910	(O-2)
Eigenvalues of the System	+0.000380	±j120.912	$+5.302285\times10^{-8}$	±j120.912	(S-4)
	-0.191068	±j61.899	-0.000001	±j61.701	(S-3)
	-1.076455	0.	-1.079741	0.	(C-1)
	-7.512587	±j21.594	-1.379978	±j21.925	(C-2)
	-33.593440	0.	-33.014635	0.	(C-1)
	-0.991241	±j3.618	-0.923314	±j3.591	(O-1)
	-13.445658	±j13.490	-13.493854	±j15.910	(O-2)

Two Controlled, Two Suppressed Modes

Sensor Locations	0.2	0.4	(S-x) - (Suppressed-Mode) Eigenvalues
Actuator Locations	0.6	0.9	(O-x) - (Observer-Mode) Eigenvalues
Singular Values of $\underline{B}_s$	1.76	0.000043	(C-x) - (Controlled-Mode) Eigenvalues

Uncontrolled Structural Damping Ratio = 0.0

than actuators, could not be chosen such that one of the non-zero singular values of $\underline{B}_s$ would go to zero. However, actuator placement could be chosen to minimize a non-zero singular value. The non-linear behavior of this singular value is demonstrated in Figure 5 which represents three of the actuator placement combinations used. The figure was generated setting one actuator at a specific location and varying the location of the second actuator. The curves represent the minimum singular value found using all possible actuator locations with a step size of 0.05 from 0.0 to 1.0. The curves also demonstrate that the only actuator combinations that can produce a zero singular value, indicating a reduction in the rank of $\underline{B}_s$, is where the actuators are co-located, representing a trivial solution. The minimum value of these curves will provide the best, in the least squares sense, actuator combination to be used with the method. Results obtained in this case are presented in Tables III and IV. Table III represents the results obtained using the actuator combination (0.95, 0.79), which produced the smallest singular value (0.16), and Table IV, an actuator combination of (0.65, 0.90), which produced a singular value of 0.265. The third actuator combination (0.90, 0.66) produced results very similar to those in Table IV and was not presented. The spectral radius of $\underline{B}_s\underline{G}$ is the square root of the sum of the squares of the elements in $\underline{B}_s\underline{G}$. Both systems shown did very well in controlling and suppressing the modes required. Also, uncoupling of system eigenvalues was very good. It can be seen when comparing these two tables that

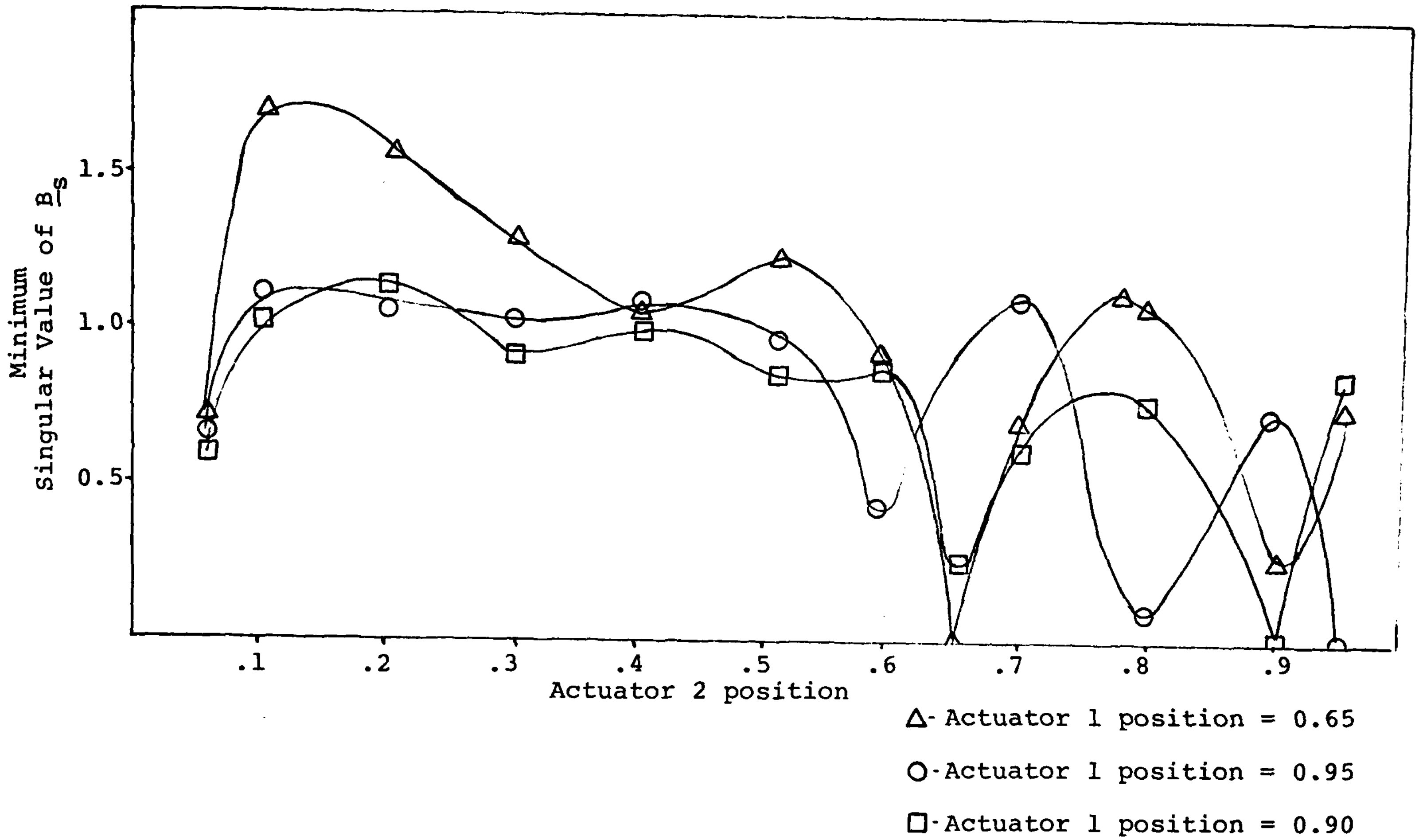

Fig 5. Singular Value of $\underline{B}_s$, 2 Actuators, 3 Suppressed Modes

Table III

System Response Using the Transformation Method

With a Minimum Singular Value for $\underline{B}_s$ of 0.16

Eigenvalues of $\underline{A}_c + \underline{B}_c G$	-1.2101	±j3.334	(C-1)
	-0.5948	±j22.022	(C-2)
	-0.3755	±j61.701	(C-3)
Eigenvalues of $\underline{A}_c - \underline{K}\ \underline{C}_c$	-6.8487	0.	(O-1)
	-2.3939	0.	(O-1)
	-16.4721	±j15.624	(O-2)
	-31.5344	±j39.903	(O-3)
Eigenvalues of the System	-1.4926	±j298.525	(S-6)
	-0.9993	±j199.852	(S-5)
	-0.6039	±j120.909	(S-4)
	-1.2101	±j3.334	(C-1)
	-0.5948	±j22.022	(C-2)
	-0.3755	±j61.201	(C-3)
	-6.8494	0.	(O-1)
	-2.3937	0.	(O-1)
	-16.4707	±j15.625	(O-2)
	-31.5413	±j39.905	(O-3)

Three Controlled, Three Suppressed Modes

Sensors/Actuators Co-located at 0.95 0.79
Uncontrolled Structural Damping Ratio = 0.05

(C-x) - (Controlled-Mode) Eigenvalues
(O-x) - (Observer-Mode) Eigenvalues
(S-x) - (Suppressed-Mode) Eigenvalues

Control Spillover, $\underline{B}_s G$, Largest Term = 0.149
Spectral Radius of $\overline{\underline{B}_s G}$ = 0.280

Table IV

System Response Using the Transformation Method

With a Minimum Singular Value for $\underline{B}_s$ of 0.265

Eigenvalues of $\underline{A}_c + \underline{B}_c\underline{G}$	-1.056	±j3.377	(C-1)
	-0.159	±j22.033	(C-2)
	-0.330	±j61.699	(C-3)
Eigenvalues of $\underline{A}_c - \underline{K}\ \underline{C}_c$	-3.173	±j1.365	(O-1)
	-19.417	±j21.484	(O-2)
	-25.596	±j38.826	(O-3)
Eigenvalues of the System	-1.492	±j298.525	(S-6)
	-0.999	±j199.852	(S-5)
	-0.602	±j120.910	(S-4)
	-1.056	±j3.377	(C-1)
	-0.159	±j22.033	(C-2)
	-0.330	±j61.694	(C-3)
	-3.173	±j1.366	(O-1)
	-19.416	±j21.483	(O-2)
	-25.597	±j38.828	(O-3)

Three Controlled, Three Suppressed Modes

Sensors/Actuators Co-located at 0.65 0.90
Uncontrolled Structural Damping Ratio = 0.05

(C-x) - (Controlled-Mode) Eigenvalues
(O-x) - (Observer-Mode) Eigenvalues
(S-x) - (Suppressed-Mode) Eigenvalues

Control Spillover, $\underline{B}_s\underline{G}$, Largest Term = 0.223
Spectral Radius of $\overline{\underline{B}_s\underline{G}}$ = 0.314

control spillover is less and the system responds better as the singular value decreased. These effects were verified through further testing.

Observation Spillover Reduction

This section uses the Transformation Method to reduce observation spillover. Since the Transformation Method apparently worked so well in minimizing control spillover, the method was applied to reduce observation spillover with the thought that system response may be further improved. In the cases tested, the method was applied to reduce both observation and control spillover simultaneously. Again, two co-located sensor/actuator pairs were used to control three modes and suppress three. The procedure was identical to that used in reducing control spillover, and was simplified due to the fact that the sensors and actuators were co-located. The results obtained proved interesting in that, although observation spillover was decreased, the method actually appeared to decrease the overall stability of the system. In the cases tested, all system eigenvalues remained essentially the same, except for three. These three were observer eigenvalues associated with the $\underline{A}_c - \underline{K}\,\underline{C}_c$ matrix. A representative example illustrating this effect is shown in Table V, where control spillover is reduced in both cases, but observation spillover in only one. The eigenvalues of the three suppressed modes, which are directly influenced by the combined effect of observation and control spillover, are not significantly

Table V

Effect of Minimizing Observation Spillover

	System With Observation Spillover		System With Observation Spillover Minimized		
Eigenvalues of $\underline{A}_c + \underline{B}_c\underline{G}$	-1.210	±j3.334	-1.210	±j3.334	(C-1)
	-0.595	±j22.036	-0.594	±j22.022	(C-2)
	-0.376	±j61.700	-0.375	±j61.701	(C-3)
Eigenvalues of $\underline{A}_c - \underline{K}\,\underline{C}_c$	-2.394	0.	-2.409	0.	(O-1)
	-6.848	0.	-9.025	0.	(O-1)
	-16.472	±j15.626	-9.869	±j13.276	(O-2)
	-31.539	±j39.903	-2.415	±j60.478	(O-3)
Eigenvalues of the System	-1.492	±j298.525	-1.492	±j298.525	(S-6)
	-0.999	±j199.852	-0.999	±j199.852	(S-5)
	-0.604	±j120.910	-0.604	±j120.910	(S-4)
	-1.210	±j3.068	-1.210	±j3.334	(C-1)
	-0.595	±j22.036	-0.594	±j22.022	(C-2)
	-0.376	±j61.700	-0.375	±j61.701	(C-3)
	-2.393	0.	-2.409	0.	(O-1)
	-6.848	0.	-9.024	0.	*(O-1)
	-16.472	±j15.624	-9.869	±j13.276	*(O-2)
	-31.539	±j39.905	-2.415	±j60.478	*(O-3)

Three Controlled, Three Suppressed Modes

Sensors/Actuators Co-Located at 0.95 0.79
Uncontrolled Structural Damping Ratio = 0.05

(C-x) - (Controlled-Mode) Eigenvalues
(O-x) - (Observer-Mode) Eigenvalues
(S-x) - (Suppressed-Mode) Eigenvalues

* Observer Eigenvalues that changed considerably when Observation Spillover was minimized.

changed, however, the eigenvalues of the $\underline{A}_c$ - $\underline{K}$ $\underline{C}_c$ matrix, an indication of the rate at which the observer error terms decay, have generally shifted to the right. Evidently, the Transformation Method, applied in reducing observation spillover, reduces observability, much as it reduces control, in achieving a reduction in spillover. This reduction in observability has the effect of producing a gain matrix, $\underline{K}$, that moves the eigenvalues of $\underline{A}_c$ - $\underline{K}$ $\underline{C}_c$ closer to the unstable region. From these results it appears that applying this method to reduce the observation spillover in a system would degrade system response.

Sensitivity of System to Parameter Variation

This final portion of the investigation introduces mode shape errors to determine system sensitivity. Systems with a stable versus an unstable controller are examined. The system considered used two sensors and two actuators to control the first two modes and suppress modes three and four. The Transformation Method was applied and errors in modal amplitude, evaluated at the actuator locations, were simulated up to ±20% in 5% increments. Although a great deal of data was generated for systems with either a stable or an unstable controller, no definite determination could be made that an unstable controller produced a system that was more susceptable to modal errors. It was noted, however, that as the simulated error increased, the uncoupling of system eigenvalues degraded significantly more in a system with an unstable controller, than in one with a stable one. Table VI represents the results typically obtained in this analysis. The effect of a 20% simulated error for a system with a stable controller is compared to a system using an unstable controller. The same analysis was conducted on a system controlling the first three modes and suppressing modes four, five, and six, producing similar results. Although the eigenvalues of the system did not vary as significantly as the uncoupling effect in this model, it is possible that this uncoupling sensitivity could adversely affect a system representing a more complex model.

Table VI

Uncoupling Differences for a Stable and Unstable Controller

	Stable Controller		% Difference	Unstable Controller		% Difference	
Eigenvalues of $\underline{A}_c + \underline{B}_c\underline{G} - \underline{K}\,\underline{C}_c$	-28.249	±j22.8		-16.145	±j22.9		
	-3.002	±j9.0		-33.897	0.		
				+2.680	0.		
Eigenvalues of $\underline{A}_c + \underline{B}_c\underline{G}$	-1.29418	0.		-1.29418	0.		(C-1)
	-17.46795	0.		-17.46795	0.		(C-1)
	-5.07557	±j21.0		-5.07557	±j21.0		(C-2)
Eigenvalues of $\underline{A}_c - \underline{K}\,\underline{C}_c$	-1.79259	±j3.2		-15.59315	0.		(O-1)
	-11.69719	±j17.3		-1.62858	0.		(O-1)
				-7.36552	±j13.1		(O-2)
Eigenvalues of the System	-0.60296	±j120.9		-0.60300	±j120.9		(S-4)
	-0.33469	±j61.6		-0.29987	±j61.7		(S-3)
	-1.29391	0.	0.02	-1.26917	0.	1.9	(C-1)
	-17.07644	0.	2.2	-19.87668	0.	13.8	(C-1)
	-5.05208	±j20.9	0.5	-5.08938	±j21.1	0.2	(C-2)
	-1.78748	±j3.2	0.3	-13.60720	0.	12.7	(O-1)
	-11.89708	±j17.7	1.7	-1.66190	0.	2.0	(O-1)
				-7.14635	±j13.1	2.9	(O-2)

Two Controlled, Two Suppressed Modes

Uncontrolled Structural Damping Ratio = 0.05
Stable Controller - Sensors/Acutators
Co-located at 0.4 0.699
Unstable Controller produced by Co-locating
Sensors at 0.9 0.9
Actuators at 0.4 0.699

(C-x) - (Controlled-Mode) Eigenvalues
(O-x) - (Observer-Mode) Eigenvalues
(S-x) - (Suppressed-Mode) Eigenvalues
%Difference Determined for Real Part of Eigenvalues

<u>Conclusions</u>

The two main conclusions to be drawn from this report are that the destabilizing effect, caused through interaction of observation and control spillover, on suppressed mode response can effectively be reduced, and that the system examined was quite insensitive to modal error.

Singular Perturbation Optimal Control did not yield results as encouraging as those found using the Transformation Method, however, the method was effective in reducing control spillover. Problems arise in matrix inversion, due to computer roundoff error, when weightings used become very large, and an infinite weighting would theoretically be required to completely eliminate control spillover effects. Only in this limiting case does the Singular Perturbation Method yield results comparable to those obtained using the Transformation Method with appropriate placement of actuators.

The Transformation Method was found to be very effective in eliminating control spillover and uncoupling of system eigenvalues, when the number of actuators in the system was equal to/or greater than the number of suppressed modes. Where the number of modes to be suppr ssed exceeded the number of actuators in the system, the method provided the best, in a least squares sense, actuator locations to minimize control spillover.

Reduction of observation spillover using the Transformation Method was found to degrade system response. The reduction of observability, an adverse effect of decreasing the observation spillover, apparantly was sufficient to alter the observation gain matrix, such that the eigenvalues of $\underline{A}_C - \underline{K}\,\underline{C}_C$, which indicate the rate of decay in observation error, shifted to the right, producing a destabilizing effect on system response.

The modal error analysis conducted did not substantiate a presupposition that a system with an unstable controller would be more sensitive to parameter variation than one with a stable one. In both cases, it was found that the system was quite insensitive to modal error, with the eigenvalues of the system never differing greater than the errors induced. Data generated, however, did show that the uncoupling of system eigenvalues was significantly more sensitive to parameter variation in a system with an unstable controller. Although this uncoupling sensitivity did not affect system response in the model tested, it remains to be determined if it would cause problems in a more complex model.

Recommendations

In a large complex structure the number of modes to be controlled and suppressed could become quite large. As the number of modes considered in a model increases, the computer capabilities as well as sensor and actuator requirements to control the system would have to increase. The first concern, therefore, in modelling a structure, is which modes should be considered critical and included in the system. A method of determining these critical modes, in a complex structure, would have to be the first step in implementing the control methods. Further study, of the methods presented in this thesis, on a complex model is needed to determine the viability of this approach to controlling large space structures. Unstable controllers in a complex model should also be examined for sensitivity to modelling inaccuracies, as the analysis presented could not dismiss this possibility.

Bibliography

1. Meirovitch, L. Elements of Vibrational Analysis. New York: McGraw-Hill, Inc., 1975.

2. Balas, M. J. "Active Control of Flexible Systems." AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft, Blacksburg, June 14, 1977.

3. Sesak, J. R. "Control of Large Space Structures Via Singular Perturbation Optimal Control." AIAA Conference on Large Space Platforms: Future Needs and Capabilities, Los Angeles, California, September 27-29, 1978.

4. Coradetti, T. "Orthogonal Subspace Reduction of Optimal Regulator Order," General Dynamics Convair Division, San Diego, California, 1979.

5. Hungerford, J. B. "Active Control of Bending Vibration in a Cantilever Beam. ", Unpublished MS Thesis, School of Engineering, Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio, December 1977.

6. Strang, G. Linear Algebra and Its Applications. New York: Academic Press, 1976.

7. Johnson, C. D. "State-Variable Design Methods May Produce Unstable Feedback Controllers.", International Journal of Control, Vol. 29, No. 4: 607-619, 1979.

Appendix A

Main Computer Program Listing

```
      PROGRAM LUOBS2(INPUT=/80,OUTPUT=/132,TAPE5,TAPE6,TAPE7)
      DIMENSION BL(20),G(2,20),C(2,20),B(6,2),WNAT(20),Z(6,6),WK(99)
     %,APRIME(20,20),Q(50,50),W(50),AP(50,50),WT(6,6),X(6,6),S(6,6),
     %AK(20,2),ZA(20,20),ZB(20,20),AD(20,20),COM1(20,20),COM2(20,20),
     %CR(2,10),BR(6,10),AR(10,10),H12(20,10),H21(10,20),ZZ(20,20),
     %ADAMP(20,20),ADAMPR(20,20),A(6,6),BRSIN(6,2),BIN(6,6),KT(2,6),
     %SIN(4),T1(2,1),R(2,2),RIN(2,2),G1(1,6),ROBIN(2,2),ROB(2,2),BT(2,6)
     %,C1(6,2),CTR(6,2),CTRC(6,6),FOB(6,6),XOB(6,6),ZOB(6,6),RC(2,6),
     %RT(1,1),T1T(1,2),T1TR(1,2),B1T(6,1),RTBT(1,6),RTINV(1,1),B1TT(1,6)
     %,Z5(6,6),BX(6,2),BRX(6,2),CX(2,6),CRX(2,6),BRIN(6,2),T2(2,1)
     %,CIN(6,6),CRSIN(6,2),SIN2(4),T2TR(1,6),KT1(1,6),CRT(6,2),T2T(1,2)
      COMMON/SHAPE/L,M
      COMMON/FUNCT/APRIME,NF
      COMMON/MAIN1/NDIM,NDIM1,COM1
      COMMON/INOU/KIN,KOUT,KPUNCH
      COMMON/MAIN2/COM2
      EXTERNAL FN
      REAL L,M
      REAL KT
      COMPLEX W,Q
      DATA IB/6/,IS/6/,IX/6/,IG/2/,IZ/6/,IADP/20/,IQ/50/,IZZ/20/,
     %IZA/20/,IBR/6/,IH21/10/,IAP/50/,IAK/20/,IC/2/,ICR/2/,IH12/20/,
     %IZB/20/,IAR/10/,IARINV/10/,IAIBR/10/
      NC=1
      NR=3
      N=3
      NC2=2*NC
      NR2=NR*2
      NT=NR+N
      N2=NDIM=2*N
      NF=2*N2+NR2
      NFO=2*N2
      NDIM1=NDIM+1
      ZETA=0.005
      INIT=0
    1 READ*,L,M,PAIR1,PAIR2,PAIR3,PAIR4,(BL(I),I=1,NT),EI
      IF(EOF(5LINPUT).NE.0.) STOP "GRACEFULLY"
      READ*, (WT(I,I),I=1,N2)
      DO 5 I=1,N2
      DO 5 J=1,N2
      FOB(I,J)=0.
    5 CONTINUE
      READ*,(FOB(I,I),I=1,N2)
      R(1,1)=R(2,2)=ROB(1,1)=ROB(2,2)=1.
      R(1,2)=R(2,1)=ROB(1,2)=ROB(2,1)=0.
      DO 3 I=1,NT
    3 WNAT(I)=((BL(I)/L)**2)*SQRT(EI/M)
      DO 4 I=1,N2
      DO 4 J=1,N2
      A(I,J)=0.
      AD(I,J)=0.
      ADAMP(I,J)=0.
      AR(I,J)=0.
      ADAMPR(I,J)=0.
      CONTINUE
      A(4,1)=AD(4,1)=ADAMP(4,1)=-WNAT(1)**2
      A(5,2)=AD(5,2)=ADAMP(5,2)=-WNAT(2)**2
```

```
      A(6,3)=AD(6,3)=ADAMP(6,3)=-WNAT(3)**2
      A(1,4)=AD(1,4)=ADAMP(1,4)=1.
      A(2,5)=AD(2,5)=ADAMP(2,5)=1.
      A(3,6)=AD(3,6)=ADAMP(3,6)=1.
      A(4,4)=AD(4,4)=ADAMP(4,4)=-2.*ZETA*WNAT(1)
      A(5,5)=AD(5,5)=ADAMP(5,5)=-2.*ZETA*WNAT(2)
      A(6,6)=AD(6,6)=ADAMP(6,6)=-2.*ZETA*WNAT(3)
      AR(4,1)=ADAMPR(4,1)=-WNAT(4)**2
      AR(5,2)=ADAMPR(5,2)=-WNAT(5)**2
      AR(6,3)=ADAMPR(6,3)=-WNAT(6)**2
      AR(1,4)=ADAMPR(1,4)=1.
      AR(2,5)=ADAMPR(2,5)=1.
      AR(3,6)=ADAMPR(3,6)=1.
      AR(4,4)=ADAMPR(4,4)=-2.*ZETA*WNAT(4)
      AR(5,5)=ADAMPR(5,5)=-2.*ZETA*WNAT(5)
      AR(6,6)=ADAMPR(6,6)=-2.*ZETA*WNAT(6)
      PRINT 1110, PAIR1,PAIR2,PAIR3,PAIR4,L,M,EI
      PRINT 1115, (BL(I),I=1,NT),(WNAT(I),I=1,NT)
      DO 10 I=1,N2
      B(I,1)=B(I,2)=0.
   10 CONTINUE
      DO 12 I=1,N2
      DO 12 J=1,N2
      IF(I.NE.J) WT(I,J)=0.
   12 CONTINUE
      J=N+1
      DO 15 I=J,N2
      CALL MODES(B(I,1),BL(I-N),PAIR1)
      CALL MODES(B(I,2),BL(I-N),PAIR2)
   15 CONTINUE
      DO 35 I=J,N2
      C(1,I)=C(2,I)=0.
   35 CONTINUE
      DO 37 I=1,NR2
      CR(1,I)=BR(I,1)=CR(2,I)=BR(I,2)=0.
   37 CONTINUE
      DO 40 I=1,NR
      CALL MODES(C(1,I),BL(I),PAIR3)
      CALL MODES(C(2,I),BL(I),PAIR4)
   40 CONTINUE
      DO 45 I=1,NR
      CALL MODES(BR(I+N,1),BL(I+N),PAIR1)
      CALL MODES(BR(I+N,2),BL(I+N),PAIR2)
      CALL MODES(CR(1,I),BL(I+N),PAIR3)
      CALL MODES(CR(2,I),BL(I+N),PAIR4)
   45 CONTINUE
      DO 53 I=1,NR2
      DO 53 J=1,NC2
      CRSIN(I,J)=CR(J,I)
      BRSIN(I,J)=BR(I,J)
   53 CONTINUE
      DO 54 I=1,NR2
      DO 54 J=1,NR2
      CIN(I,J)=0.
      BIN(I,J)=0.
   54 CONTINUE
      DO 56 I=1,NR2
```

```
      CIN(1,1)=1.
      BIN(1,1)=1.
   56 CONTINUE
      PRINT 1645
      PRINT*, "  F  "
      DO 61 I=1,N2
      PRINT*,(WT(I,J),J=1,N2)
   61 CONTINUE
      WRITE*, " "
      PRINT*, " R "
      DO 62 I=1,NC2
   62 PRINT*,(R(I,J),J=1,NC2)
      WRITE*, " "
      PRINT*, " FOB"
      DO 63 I=1,N2
   63 PRINT*, (FOB(I,J),J=1,N2)
      WRITE*, " "
      PRINT*, "ROB"
      DO 64 I=1,NC2
   64 PRINT*,(ROB(I,J),J=1,NC2)
      PRINT 1200
      DO 49 I=1,N2
      PRINT 1300,(ADAMP(I,J),J=1,N2)
   49 CONTINUE
      PRINT 1325
      DO 51 I=1,NR2
      PRINT 1350,(ADAMPR(I,J),J=1,NR2)
   51 CONTINUE
      PRINT 1400
      DO 58 I=1,N2
   58 PRINT 1300,(B(I,J),J=1,NC2)
      PRINT 1450
      DO 59 I=1,NR2
   59 PRINT 1350,(BR(I,J),J=1,NC2)
      PRINT 1500
      PRINT 1300,(C(1,I),I=1,N2)
      PRINT 1300,(C(2,I),I=1,N2)
      PRINT 1550
      PRINT 1350,(CR(1,I),I=1,NR2)
      PRINT 1350,(CR(2,I),I=1,NR2)
      PRINT 1575
      CALL LSVDF(BRSIN,NR2,NR2,NC2,BIN,NR2,NR2,SIN,WK,IER)
      DO 68 I=1,NC2
      WRITE*, SIN(I)
   68 CONTINUE
      DO 121 I=1,NC2
      T1(I,1)=BRSIN(I,2)
  121 T1T(1,I)=T1(I,1)
      PRINT 1580
      DO 67 I=1,NC2
   67 WRITE*,T1(I,1)
      DO 97 I=1,NR2
      DO 97 J=1,NC2
      CT(I,J)=C(J,I)
   97 CONTINUE
      CALL LINV2F(ROB,NC2,NC2,ROBIN,3,WK,IER)
      CALL VMULFF(CT,ROBIN,N2,NC2,NC2,N2,NC2,CTR,N2,IER)
```

```
      CALL  VMULFF(CTR,C,N2,NC2,N2,N2,NC2,CTRC,N2,IER)
      CALL  MRIC(N2,A,CTRC,FOF,XOB,ZOB,MR,INIT)
      CALL  VMULFF(ROBIN,C,NC2,NC2,N2,NC2,NC2,RC,NC2,IER)
      CALL  VMULFF(RC,XOB,NC2,N2,N2,NC2,N2,KT,NC2,IER)
      WRITE*, " "
      PRINT  2000
      DO 98  I=1,NC2
   98 PRINT  1300, (KT(I,J),J=1,N2)
      DO 122 I=1,NR2
      DO 122 J=1,NR2
      S(I,J)=0.
  122 CONTINUE
      CALL  VMULFF(B,T1,NR2,NC2,NC,NR2,NC2,B1T,NR2,IER)
      DO 161 I=1,NR2
      B1TT(1,I)=B1T(I,1)
  161 CONTINUE
      CALL  VMULFF(T1T,R,NC,NC2,NC2,NC,NC2,T1TR,NC,IER)
      CALL  VMULFF(T1TR,T1,NC,NC2,NC,NC,NC2,RT,NC,IER)
      CALL  LINV2F(RT,NC,NC,RTINV,0,WK,IER)
      CALL  VMULFF(B1T,RTINV,NR2,NC,NC,NR2,NC,BRIN,NR2,IER)
      CALL  VMULFF(BRIN,B1TT,NR2,NC,NR2,NR2,NC,S,NR2,IER)
      INIT=0
      CALL  MRIC(NR2,A,S,WT,X,Z,MR,INIT)
      IF(MR.EQ.-1) STOP "MRIC DID NOT CONVERGE"
      CALL  VMULFF(RTINV,B1TT,NC,NC,NR2,NC,NC,RTBT,NC,IER)
      CALL  VMULFF(RTBT,X,NC,NR2,NR2,NC,NR2,G1,NC,IER)
      IF(IER.EQ.129) STOP "BT*X BAD"
      CALL  VMULFF(T1,G1,NC2,NC,NR2,NC2,NC,G,NC2,IER)
      PRINT  1600
      PRINT  1300,(G(1,I),I=1,N2)
      PRINT  1300,(G(2,I),I=1,N2)
      CALL  VMULFF(B,G,N2,NC2,N2,IB,IG,Z,IZ,IER)
      IF(IER.EQ.129) STOP "B*G BAD"
      DO 102 I=1,N2
      DO 102 J=1,NC2
  102 AK(I,J)=KT(J,I)
      CALL  VMULFF(AK,C,N2,NC2,N2,IAK,IC,Z3,IZ3,IER)
      IJOB=0
      CALL  EIGRF(ADAMP,N2,IADP,IJOB,W,Q,IQ,WK,IER)
      IF(IER.NE.0)  PRINT*,"IER = ",IER
      PRINT  500
      DO 57  I=1,N2
      PRINT*,W(I)
 57   CONTINUE
      DO 66  I=1,N2
      DO 66  J=1,N2
   66 CONTINUE
      PRINT  1750
      DO 80  I=1,N2
   80 PRINT  1300,(Z5(I,J),J=1,N2)
      IJOB=0
      CALL  EIGRF(Z5,N2,N2,IJOB,W,Q,IQ,WK,IER)
      PRINT  1755
      DO 81  I=1,N2
      PRINT*, W(I)
   81 CONTINUE
      PRINT  1760
```

```
      FACTOR1=0.8
      FACTOR2=1.2
      WRITE*, " "
      WRITE*, " "
      WRITE*, " "
      WRITE*, " "
      WRITE*, " "
      DO 120 IJ=1,9
      PRINT*, "FACTOR 1  =  ", FACTOR1, "  FACTOR 2  =  ", FACTOR2
      DO 114 I=1,N2
      DO 114 J=1,NC2
      BRX(I,J)=BX(I,J)=CX(J,I)=CRX(J,I)=0.
  114 CONTINUE
      BX(5,1)=FACTOR2*B(5,1)
      BX(4,1)=FACTOR1*B(4,1)
      BX(6,1)=FACTOR1*B(6,1)
      BX(6,2)=FACTOR2*B(6,2)
      BX(5,2)=FACTOR1*B(5,2)
      BX(4,2)=FACTOR2*B(4,2)
      BRX(5,1)=FACTOR2*BR(5,1)
      BRX(5,2)=FACTOR1*BR(5,2)
      BRX(6,1)=FACTOR1*BR(6,1)
      BRX(6,2)=FACTOR2*BR(6,2)
      BRX(4,1)=FACTOR1*BR(4,1)
      BRX(4,2)=FACTOR2*BR(4,2)
      CX(1,1)=FACTOR2*C(1,1)
      CX(1,2)=FACTOR1*C(1,2)
      CX(2,1)=FACTOR1*C(2,1)
      CX(2,2)=FACTOR2*C(2,2)
      CX(1,3)=FACTOR2*C(1,3)
      CX(2,3)=FACTOR1*C(2,3)
      CRX(1,1)=FACTOR2*CR(1,1)
      CRX(1,2)=FACTOR1*CR(1,2)
      CRX(2,1)=FACTOR1*CR(2,1)
      CRX(2,2)=FACTOR2*CR(2,2)
      CRX(1,3)=FACTOR2*CR(1,3)
      CRX(2,3)=FACTOR1*CR(2,3)
      CALL VMULFF(BRX,G,N2,NC2,N2,IBR,IG,H21,IH21,IER)
      IF(IER.GT.129) STOP "BR*G BAD"
      PRINT 1700
      DO 111 I=1,N2
      PRINT 1300,(H21(I,J),J=1,N2)
  111 CONTINUE
      CALL VMULFF(BX,G,N2,NC2,N2,IB,IG,Z,IZ,IER)
      DO 55 I=1,N2
      DO 55 J=1,N2
      APRIME(I,J+N2)=-Z(I,J)
      ZA(I,J)=A(I,J)-Z(I,J)
      Z(I,J)=AD(I,J)-Z(I,J)
      APRIME(I,J)=Z(I,J)
      ZZ(I,J)=Z(I,J)
      APRIME(I+N2,J)=0.
 55   CONTINUE
      IJOB=0
      CALL EIGRF(ZZ,N2,IZZ,IJOB,W,Q,IQ,WK,IER)
      IF(IER.NE.0) PRINT*,"IER = ",IER
      PRINT 1700
```

```
      DO 60 I=1,N2
      PRINT 1300,(Z(I,J),J=1,N2)
 60   CONTINUE
      PRINT 1800
      DO 65 I=1,N2
      PRINT*,W(I)
 65   CONTINUE
      CALL VMULFF(AK,CX,N2,NC2,N2,IAK,IC,ZB,IZB,IER)
      DO 100 I=1,N2
      DO 100 J=1,N2
      ZB(I,J)=AD(I,J)-ZB(I,J)
      ZA(I,J)=ZB(I,J)
      APRIME(I+N2,J+N2)=ZB(I,J)
 100  CONTINUE
      IJOB=0
      CALL EIGRF(ZA,N2,IZA,IJOB,W,Q,IQ,WK,IER)
      IF(IER.NE.0) PRINT*,"IER = ",IER
      PRINT 2200
      DO 105 I=1,N2
      PRINT 1300,(ZB(I,J),J=1,N2)
 105  CONTINUE
      PRINT 2300
      DO 110 I=1,N2
      PRINT*,W(I)
 110  CONTINUE
      CALL VMULFF(AK,CRX,N2,NC2,N2,IAK,ICR,H12,IH12,IER)
      DO 112 I=1,NR2
      DO 112 J=1,N2
      APRIME(I+12,J)=H21(I,J)
      APRIME(I+12,J+6)=H21(I,J)
      APRIME(J,I+12)=0.
      APRIME(J+6,I+12)=H12(J,I)
 112  CONTINUE
      DO 113 I=13,NF
      DO 113 J=13,NF
      APRIME(I,J)=AR(I-12,J-12)
 113  CONTINUE
      DO 115 I=1,NF
      DO 115 J=1,NF
      AP(I,J)=APRIME(I,J)
 115  CONTINUE
      PRINT 2550
      DO 99 I=1,NF
   99 PRINT 2600,(APRIME(I,J),J=1,6)
      WRITE*, " "
      WRITE*, " "
      DO 126 I=1,NF
  126 PRINT 2600,(APRIME(I,J),J=7,12)
      WRITE*," "
      WRITE*," "
      DO 127 I=1,NF
  127 PRINT 2600,(APRIME(I,J),J=13,18)
      IJOB=0
      CALL EIGRF(AP,NF,IAP,IJOB,W,Q,IQ,WK,IER)
      IF(IER.NE.0) PRINT*,"IER = ",IER
      WRITE*, " "
      WRITE*, " "
```

```
      PRINT 2400
      DO 125 I=1,NF
      PRINT*,W(I)
  125 CONTINUE
      FACTOR1=FACTOR1+0.05
      FACTOR2=FACTOR2-0.05
  120 CONTINUE
      GO TO 1
  900 FORMAT(//10X,"THE EIGENVALUES OF A"/)
 1110 FORMAT(1H1/////,25X,"CANTILEVER BEAM WITH TWO SENSORS,
     %2 ACTUATORS, 3 CONTROLLED MODES AND 3 RESIDUAL MODES",
     %//50X,"FIRST ACTUATOR POSITION=",F14.8,
     %//50X,"SECOND ACTUATOR POSITION =",F14.8,
     %//50X," FIRST SENSOR POSITION =",F14.8,
     %//50X, "SECOND SENSOR POSITION =",F14.8,
     %//50X,"BEAM LENGTH=",F5.1,
     %/50X,"BEAM MASS DENSITY/UNIT LENGTH = ",F3.6,
     %/50X,"BEAM STIFFNESS  (EI) = ",F3.2)
 1115 FORMAT(/50X,"ROOTS OF THE CHARACTERISTIC EQUATION:",
     %/55X,"FIRST MODE: ",E20.14,/55X,"SECOND MODE: ",E20.14,
     %/55X,"THIRD MODE: ",E20.14,
     %/55X,"FOURTH MODE: ",E20.14,
     %/55X,"FIFTH MODE : ",E20.14,
     %/55X,"SIXTH MODE :",E20.14,
     %/50X,"NATURAL FREQUENCIES:",
     %/55X,"FIRST MODE: ",E20.14,/55X,"SECOND MODE: ",E20.14,
     %/55X,"THIRD MODE: ",E20.14,
     %/55X,"FOURTH MODE :",E20.14,
     %/55X,"FIFTH MODE : ",E20.14,
     %/55X,"SIXTH MODE: ",E20.14)
 1200 FORMAT(//25X,* THE "A" MATRIX*/)
 1300 FORMAT(/2X,6E16.7)
 1325 FORMAT(//20X,*THE "A" MATRIX  (RESIDUAL)*/)
 1350 FORMAT(/2X,6E16.7)
 1400 FORMAT(//20X,* THE "B" MATRIX*/)
 1450 FORMAT(//15X,*THE "B" MATRIX FOR RESIDUALS*/)
 1500 FORMAT(//20X,*THE "C" MATRIX*/)
 1550 FORMAT(//20X,*THE "C" MATRIX FOR RESIDUALS*/)
 1575 FORMAT(//20X,*SINGULARITIES OF THE "B" MATRIX FOR RESIDUALS*/)
 1576 FORMAT(//20X,*SINGULARITIES OF THE C MATRIX FOR RESIDUALS*/)
 1580 FORMAT(//20X,*THE "T" MATRIX FOR BR*/)
 1581 FORMAT(//20X,*THE  "T" MATRIX FOR CR*/)
 1520 FORMAT(20X,*THE "G" MATRIX*/)
 1535 FORMAT(//20X,*THE BR TIMES T MATRIX*/)
 1645 FORMAT(//10X,*THE WEIGHTS USED*/)
 1700 FORMAT(//57X,*THE "A+BG" MATRIX*/)
 1750 FORMAT(//20X,*THE A + BG - KC MATRIX*/)
 1755 FORMAT(//10X,*THE EIGENVALUES OF A + BG - KC MATRIX*/)
 1760 FORMAT(//10X,"TO ACCOUNT FOR MODAL ERROR THE  B  AND  C"
     %//10X,"MATRICIES ARE VARIED BY FACTOR 1 FOR ODD ELEMENTS AND "
     %//10X,"BY FACTOR 2 FOR EVEN ELEMENTS"/)
 1770 FORMAT(//10X,*BR TIMES G MATRIX*/)
 1800 FORMAT(//10X,"THE EIGENVALUES OF A+BG"/)
 1900 FORMAT(10X,"THE SMALLEST REAL PART = ",E23.13/)
 2000 FORMAT(//35X,*THE "K" MATRIX TRANSPOSED*/)
 2200 FORMAT(//57X,*THE "A-KC" MATRIX*/)
 2300 FORMAT(//10X,"THE EIGENVALUES OF A-KC"/)
```

```
2400 FORMAT(//10X,"THE EIGENVALUES OF THE SYSTEM"/)
2500 FORMAT(//40X," THE SYSTEM MATRIX"/)
2600 FORMAT(/2X,6E16.7)
     END
```

```
      SUBROUTINE MODES(PHI,BLI,X)
      COMMON/SHAPE/L,M
      REAL L,M
      BX=BLI/L*X
      PHI=((SIN(BLI)-SINH(BLI))*(SIN(BX)-SINH(BX))+(COS(BLI)+COSH(BLI))*
     %(COS(BX)-COSH(BX)))/(SQRT(M*L)*SIN(BLI)*SINH(BLI))
      RETURN
      END
```

Appendix B

Singularity Computer Program Listing

```
      PROGRAM LUOBS2 (INPUT = 180, OUTPUT =/132, TAPE 5, TAPE 6, TAPE 7)
      DIMENSION BS(6,2), BL(3), BS SIN (6,2), BIN(6,6), SIN(2), WK(99)
      N=2
      NR=3
      NR2=6
      BL(1)=10.996
      BL(2)=14.137
      BL(3)=17.278
      ACT(1)=0.05
      DO 17 II=1,20
      ACT(2)=0.05
      DO 16 JJ=1,20
      DO 5 I+1,3
      CALL MODES(BS(I,1),BL(I),ACT(1))
      CALL MODES(BS(I,2),BL(I),ACT(2))
    5 CONTINUE
      DO 6 I=1,NR2
      DO 6 J=1,N
      BSSIN(I,J)=BS(I,J)
    6 CONTINUE
      DO 7 I=NR2
      DO 7 J=NR2
      BIN(I,J)=0.
    7 CONTINUE
      DO 8 I=1,NR2
      BIN(I,I)=1.
    8 CONTINUE
      CALL LSVDF(BSSIN,NR2,NR2,N,BIN, NR2,NR2,SIN,WK,IER)
      WRITE*,"SINGULARITIES OF BS"
      DO 9 I=1,N
      WRITE*, SIN(I)
    9 CONTINUE
      ACT(2)=ACT(2)+.05
   16 CONTINUE
      ACT(1)=ACT(1)+.05
   17 CONTINUE
      STOP
      END
```

Vita

Keith Daniel Sanborn was born on June 22, 1948 in Boston, Massachusetts. He graduated from high school in Taunton, Massachusetts, in 1966, and attended Wentworth Institute, Boston, Massachusetts, from which he received an Associates Degree in Astronautical Engineering, in June, 1968. He enlisted in the Air Force on November 1, 1968, was selected into the Airman's Education and Commissioning Program, and was sent to Auburn University, Alabama, in January, 1970. He graduated with a Bachelors Degree in Astronautical Engineering in June, 1972. After attending Officers Training School, Lackland AFB, Texas, he was assigned to Williams AFB, Arizona, for Pilot Training. Graduating in October, 1973, he was assigned flying duties on a C-141 at McGuire AFB, New Jersey, where he served until entering the School of Engineering, Air Force Institute of Technology, in June, 1978.

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The equations of motion for a cantilever beam in bending vibration are developed in state vector form using a normal mode approximation. A linear optimal control system generates a feedback control proportional to the state which is represented by modal amplitudes and velocities determined using position information from sensors. The observer gain matrix and the feedback control gain matrix are both determined from a steady state

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) optimal regulator which minimizes the related quadratic performance index. Control is applied through point force actuators. The effects on beam response of sensor and actuator location, reduced observation and control spillover, restructured control performance index, and a reduced order optimal regulator are examined. Parameter variations of the modal amplitudes were tested to note effects on system stability with both stable and unstable controllers. Singular perturbation theory is used to reconstruct the control performance index, essentially adding a penalty function against any control vector that lies in the suppressed modes subspace. Singular value decomposition is used to construct a transformation matrix through which observation and/or control spillover may be eliminated. A means by which optimal actuator locations may be determined is also presented.

System response is shown to be very sensitive to actuator location. Singular perturbation provided a method through which control spillover could be minimized for the actuator locations chosen, however, it did not provide the means by which the actuators could be positioned such that the spillover effect could be eliminated. Optimal actuator locations could be found using singular value decomposition. Control and observation spillover effects could then be completely eliminated only where the number of actuators was equal to or greater than the number of modes to be suppressed. Robustness of system response to modal amplitude errors was very good, and the unstable controllers tested did not appear to seriously affect this robustness.